

Dielectric Waveguide

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Modes

Two field
components
onlyHelmholtz
equationFinite
Difference
Method

define

digitize

helmholtz

solve and
visualize

More

- permittivity $\epsilon = \epsilon(x, y)$ does not depend on z
- wave vector along z axis
- all fields are of the form
$$F(t, x, y, z) = F(x, y) e^{i\beta z} e^{-i\omega t}$$

- β is the propagation constant of the mode
- there are two kind of modes:
- quasi TE - Transversal Electric
- quasi TM - Transversal Magnetic
- they have different propagation constants
- no analytic solution

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- recall the general mode equation

$$\mathbf{curl} \mathbf{curl} \mathbf{E} = k_0^2 \epsilon(x, y) \mathbf{E}$$

- the curl operator is

$$\begin{pmatrix} 0 & -i\beta & \partial_y \\ i\beta & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{pmatrix}$$

- apply it twice

$$\begin{pmatrix} \beta^2 - \partial_y^2 & \partial_x \partial_y & i\beta \partial_x \\ \partial_x \partial_y & \beta^2 - \partial_x^2 & i\beta \partial_y \\ i\beta \partial_x & i\beta \partial_y & -\partial_x^2 - \partial_y^2 \end{pmatrix} \mathbf{E} = k_0^2 \epsilon(x, y) \mathbf{E}$$

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- problem: β and β^2
- problem: two polarization states, three fields
- divergence of $\epsilon \mathbf{E}$ vanishes
- $-\mathrm{i}\beta E_z = \epsilon^{-1} \partial_x \epsilon E_x + \epsilon^{-1} \partial_y \epsilon E_y$
- now the mode equation contains only two fields

$$\begin{pmatrix} k_0^2 \epsilon + \partial_x \epsilon^{-1} \partial_x \epsilon + \partial_y^2 & \partial_x \epsilon^{-1} \partial_y \epsilon - \partial_x \partial_y \\ \partial_y \epsilon^{-1} \partial_x \epsilon - \partial_y \partial_x & k_0^2 \epsilon + \partial_x^2 + \partial_y \epsilon^{-1} \partial_y \epsilon \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \beta^2 \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- and it is a normal eigenvalue problem!
- analogous form for magnetic fields

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- If waveguides are broad: $\partial_y \epsilon \approx \epsilon \partial_y$
- $E_x \approx 0$
- this results in the quasi TE mode equation
$$\{\partial_x^2 + \partial_y^2 + k_0^2 \epsilon(x, y)\} E_y = \beta^2 E_y$$
- Helmholtz equation
- with $\partial_y \epsilon \approx \epsilon \partial_y$ and $H_x \approx 0$
- quasi TM mode equation
$$\{\epsilon \partial_x \epsilon^{-1} \partial_x + \partial_y^2 + k_0^2 \epsilon(x, y)\} H_y = \beta^2 H_y$$
- compared with a planar waveguide: $\epsilon = \epsilon(x, y)$ and additional ∂_y^2
- and: quasi modes have z -components

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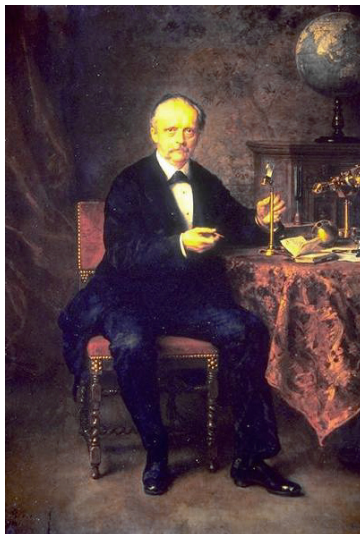
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Hermann von Helmholtz, German physicist, 1821 - 1894

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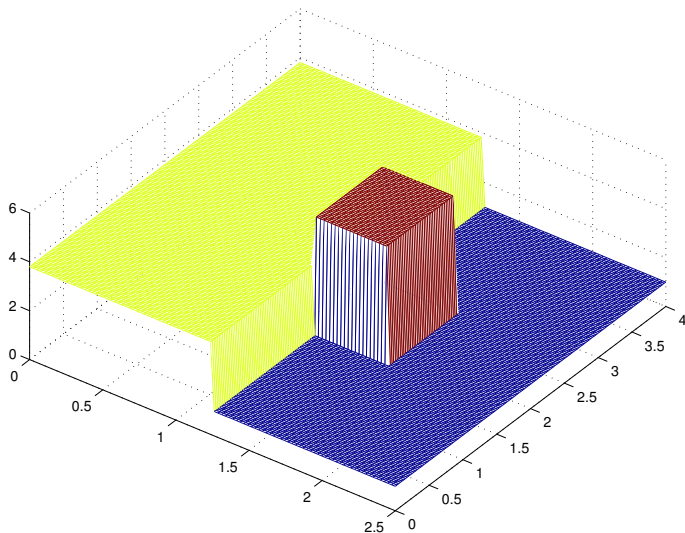
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- on top of a substrate there is a rib of enhanced permittivity
- cross section represented by $(x_i, y_k) = (ih_x, kh_y)$ with integer numbers i, k
- permittivity $\epsilon(x_i, y_k)$ represented by a matrix `eps`
- define waveguide `rwg`
- digitize it
- setup Helmholtz matrix
- solve for guided modes
- visualize result

```
1  function rwg=define
2  clear all;
3  % all lengths in microns
4  rwg.LAMBDA=1.30;
5  rwg.EC=1.00; % cover permittivity
6  rwg.ES=3.80; % substrate permittivity
7  rwg.ER=5.80; % rib permittivity
8  % computational window, [xlo,xhi,ylo,yhi]
9  rwg.CW=[0.0,2.5,0.0,4.0];
10 % rib, [xlo,xhi,ylo,yhi]
11 rwg.RB=[1.25,1.75,1.5,2.5];
12 end % function define
```



Permittivity profile of a rib waveguide

```
1 function rwg=digitize(r,NX,NY)
2 x=linspace(r.CW(1),r.CW(2),NX);
3 y=linspace(r.CW(3),r.CW(4),NY);
4 [X,Y]=meshgrid(x,y);
5 RIB=(X>r.RB(1))&(X<r.RB(2))&(Y>r.RB(3))&(Y<r.RB(4));
6 SUB=(X<=r.RB(1));
7 COV=(X>r.RB(1))&~RIB;
8 prm=r.EC*COV+r.ES*SUB+r.ER*RIB;
9 rwg=r;
10 rwg.x=x;
11 rwg.y=y;
12 rwg.eps=prm;
13 % note that NX and NY may be reconstructed
14 % from rwg.eps
15 end % function digitize
```

```
1 function rwg=helmholtz(r)
2 k0=2*pi/r.LAMBDA;
3 hx=r.x(2)-r.x(1); % x step width
4 hy=r.y(2)-r.y(1); % y step width
5 [NX,NY]=size(r.eps'); % size of computational window
6 NU=NX*NY; % number of unknowns
7 prrm=reshape(r.eps',NU,1); % permittivity
8 one=ones(NU,1);
9 md=-2*one*(1/hx^2+1/hy^2)/k0^2+prrm; % main diagonal
10 xd=one/hx^2/k0^2; % side diagonal xx differentiation
11 yd=one/hy^2/k0^2; % side diagonal yy differentiation
12 H=spdiags([yd,xd,md,xd,yd],[-NX,-1,0,1,NX],NU,NU);
13 % note that there are false links, remove
14 for n=NX:NX:NU-NX
15     H(n,n+1)=0;
16     H(n+1,n)=0;
17 end;
18 rwg=r;
19 rwg.H=H;
20 end % function helmholtz
```

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```
1  function rwg=solve(r)
2  % calculate 6 eigenvectors of sparse matrix
3  % algebraically largest eigenvalues
4  [efun,eval]=eigs(r.H,6,'la');
5  eval=diag(eval);
6  % guided modes
7  guided=(r.ES<eval);
8  modes=efun(:,guided);
9  [NX,NY]=size(r.eps);
10 rwg=r;
11 % modes must be reshaped
12 rwg.mode=reshape(modes,[NY,NX,sum(guided)]);
13 end % function solve
```

```
1 function visualize(r,m,fn)
2 close;
3 [~,h0]=contour(r.y,r.x,r.mode(:, :,m),48);
4 hold on;
5 h1=plot([r.CW(3),r.CW(4)], [r.RB(1),r.RB(1)]);
6 h2=plot([r.RB(3),r.RB(3)], [r.RB(1),r.RB(2)]);
7 h3=plot([r.RB(4),r.RB(4)], [r.RB(1),r.RB(2)]);
8 h4=plot([r.RB(3),r.RB(4)], [r.RB(2),r.RB(2)]);
9 hold off;
10 axis equal;
11 set([h0,h1,h2,h3,h4], 'LineWidth',1.5);
12 if ~strcmp(fn, '')
13     epsfn=[fn, '.eps'];
14     print('-depsc',epsfn);
15     command=['bash ', 'epstopdf ',epsfn];
16     system(command);
17     delete(epsfn);
18 end;
19 end % function visualize
```

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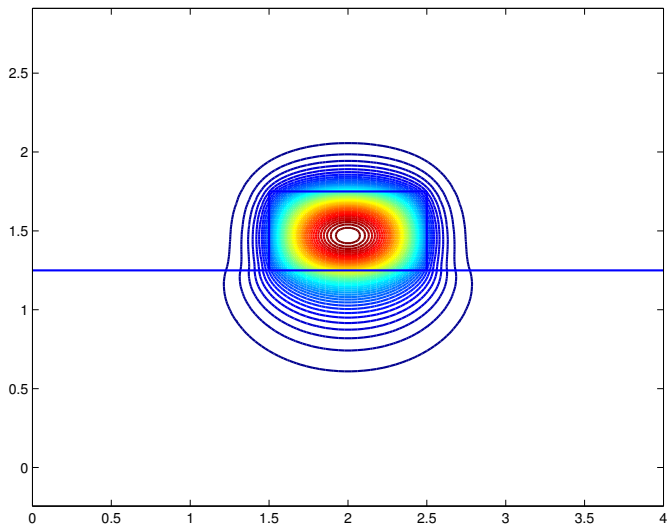
define

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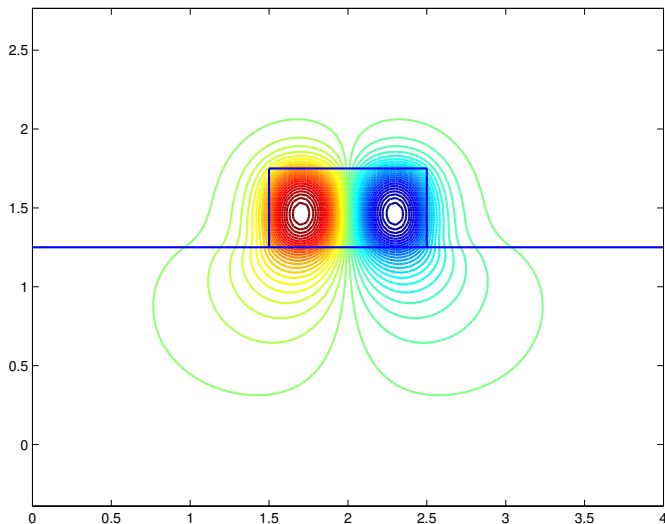
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The base mode of a model rib waveguide.



Excited mode. Note that we have plotted the amplitude.

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```
1      LAMBDA: 0.6330
2      EC: 1
3      ES: 3.8000
4      ER: 5.2000
5      CW: [0 2.5000 0 4]
6      RB: [1.2500 1.7500 1.5000 2.5000]
7      x: [1x200 double]
8      y: [1x200 double]
9      eps: [200x200 double]
10     H: [40000x40000 double]
11     mode: [200x200x5 double]
```

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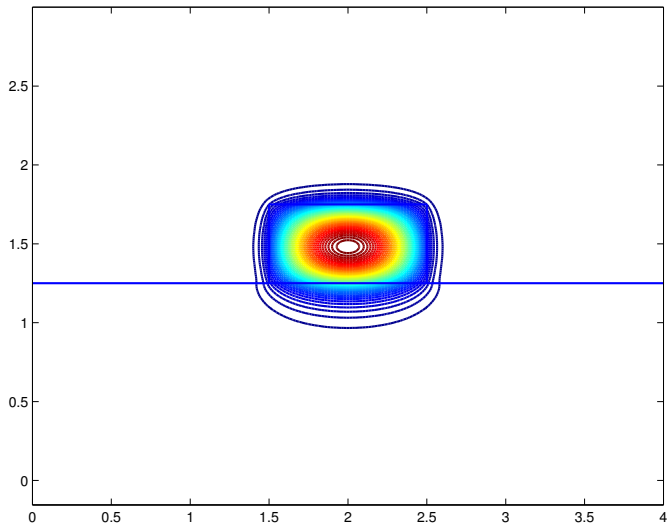
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digitize

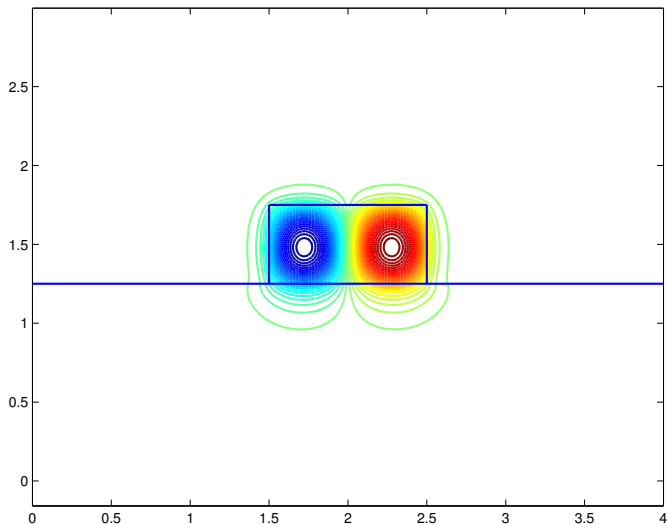
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Its base mode, TE₀

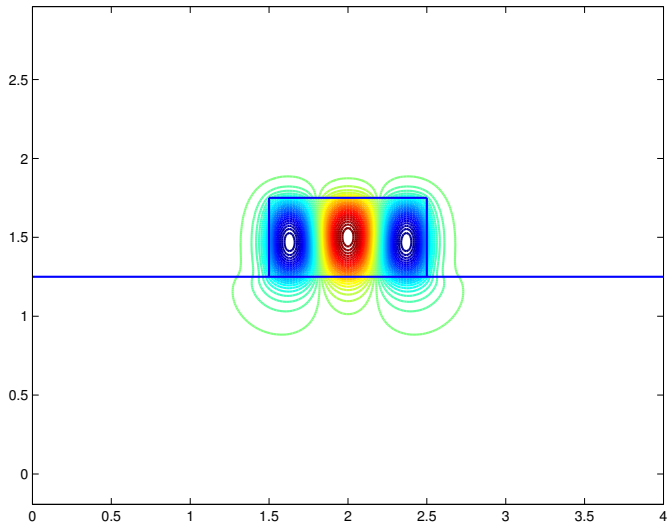


Its first excited mode, TE₁

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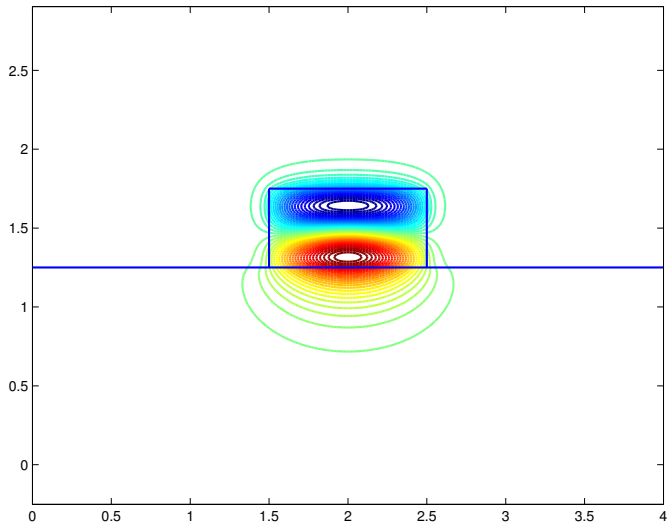
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TE3

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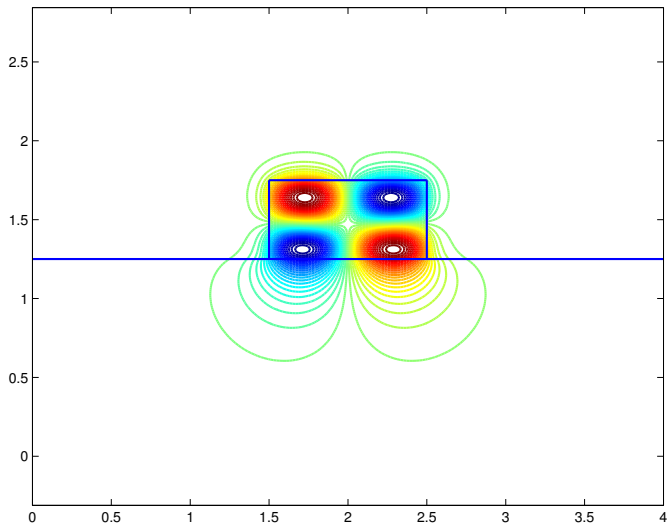
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TE4